Correspondence between the 3-form and a non-minimal multiplet in supersymmetry

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Abstract

In analogy to the chiral-linear multiplet correspondence we establish a relationship between the 3-form (or gaugino condensate) multiplet and a coupled non-minimal $(0, 1/2)$ multiplet, illustrated by a simple explicit example.

CPT-P92-2007
PCCF-RI-0705

February 2, 2008

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1 Introduction

The scalar multiplet [1], commonly termed chiral multiplet, is the most popular realization of the $(0, 1/2)$ representation (massive or massless) of supersymmetry in terms of local quantum fields. It contains as components a complex scalar, a Majorana spinor and a complex scalar auxiliary field. Another realization of the same representation (generally massless in this case) is provided by the linear multiplet [2], given in terms of a real scalar, a 2-index antisymmetric tensor gauge field, a Majorana spinor and no auxiliary field. Contrary to the previous one the linear multiplet is a gauge multiplet. In classical Lagrangian field theory one can establish [3] a certain correspondence between the chiral multiplet and the linear multiplet, sometimes referred to as *chiral-linear multiplet duality*, in particular in applications where the linear multiplet incarnates a *dilaton-axion multiplet*. In this note we would like to draw attention to yet another couple of realizations of the $(0, 1/2)$ representation, the *3-form multiplet* [4] and a *non-minimal* $(0, 1/2)$ *multiplet* [5], including simple chiral multiplet couplings.

The 3-form multiplet made of a 3-index antisymmetric tensor gauge field, a complex scalar, a Majorana spinor and a real auxiliary field may be understood as a *further constrained chiral multiplet*. It is the basic ingredient in the context of gaugino condensation, but is also relevant in the theory of supersymmetric gauge anomalies and in the description of curvature squared terms and Chern-Simons forms in supersymmetry. The non-minimal $(0, 1/2)$ multiplet, on the other hand, is less well known. In this note, we would like to outline a relation with the 3-form multiplet in very much the same vein as the above mentioned chiral-linear correspondence. To be definite, we shall exhibit here a very simple toy model, coupling the gaugino-condensate multiplet to a single generic chiral multiplet and suggest a corresponding coupling of the non-minimal $(0, 1/2)$ multiplet.

It may be worthwhile to comment briefly on the notion of *gaugino condensate multiplet*. In a supersymmetric gauge theory the gauge field-strength tensor is promoted to a multiplet containing as superpartners the gaugino and a real bosonic auxiliary field. The corresponding gaugino superfield, denoted $W_\alpha, \bar{W}^{\dot{\alpha}}$ is chiral ($\bar{D}^{\dot{\alpha}} W_\alpha = 0, D_\alpha \bar{W}^{\dot{\alpha}} = 0$) and subject to the additional constraint $D^\alpha W_\alpha = \bar{D}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}$. Irrespectively of the mechanisms underlying *gaugino condensation*, the constraints on $W_\alpha, \bar{W}^{\dot{\alpha}}$ imply that the *condensate*

superfields $tr(W^\alpha W_\alpha), tr(\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha})$ are not only chiral but fulfill the additional condition

$$D^2\, tr(W^\alpha W_\alpha) - \bar{D}^2\ tr(\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}) = i\varepsilon_{klmn} tr\left(F^{kl}F^{mn}\right).$$

Interestingly enough, this supermultiplet can be viewed as a particular realization of a generic 3-form gauge theory in superspace with the 3-form gauge potential related to the Chern-Simons form of the Yang-Mills theory.

2 The 3-Form Multiplet

In multiplets of supersymmetry different components may be assigned different R-weights, in relation to their supersymmetry transformations and the chiral properties of their generators [6]. As it seems reasonable to assign vanishing R-weight to gauge potential components, the R-weights of their supersymmetry partners are then determined correspondingly. Precisely in the case of $C_{klm}(x)$, the 3-form gauge potential of the gaugino condensate multiplet with vanishing R-weight, the weights of the other components are dictated by supersymmetry: in units where $r(\theta) = r(\bar{D}) = +1$, $r(\bar\theta) = r(D) = -1$, the complex scalar $Y(x), \overline{Y}(x)$ has $r(Y) = +2$, $r(\overline{Y}) = -2$. The fermionic components $\eta_\alpha(x), \bar\eta^{\dot\alpha}(x)$ acquire $r(\eta) = +1, r(\bar\eta) = -1$, whereas $H(x)$, the real auxiliary field has $r(H) = 0$. Therefore, $H(x)$ may constitute by itself an R-inert supersymmetric Lagrangian in analogy with the Fayet-Iliopoulos D-term familiar in supersymmetric gauge theory.

In superfield language, the 3-form multiplet is characterized by the superfields $Y, \overline{Y}$ subject to the chirality conditions

$$\bar{D}^{\dot\alpha} Y \,=\, 0, \quad D_\alpha \overline{Y} \,=\, 0, \tag{2.1}$$

and the additional constraint

$$D^2 Y - \bar{D}^2 \overline{Y} \,=\, -8i\, \partial C(x), \tag{2.2}$$

with $\partial C = -\frac{4}{3}\epsilon^{klmn}\partial_k C_{lmn}$. These superfield relations have an interpretation as Bianchi identities in superspace geometry [4], [7]. Component fields are identified as usual by projection to lowest superfield components

$$\overline{Y}| \,=\, \overline{Y}(x), \quad \bar{D}^{\dot\alpha}\overline{Y}| \,=\, \sqrt{2}\,\bar\eta^{\dot\alpha}(x), \quad Y| \,=\, Y(x), \quad D_\alpha Y| \,=\, \sqrt{2}\,\eta_\alpha(x), \tag{2.3}$$

$$D^2Y| + \bar{D}^2\overline{Y}| \;=\; -8\,H(x), \qquad C_{lmn}| = C_{lmn}(x). \tag{2.4}$$

Supersymmetry transformations for these components read

$$\delta_\xi C_{mlk} \;=\; \frac{\sqrt{2}}{16}\left(\bar{\xi}\bar{\sigma}^n\eta - \xi\sigma^n\bar{\eta}\right)\varepsilon_{nmlk}, \tag{2.5}$$

$$\delta_\xi Y \;=\; \sqrt{2}\,\xi^\alpha\eta_\alpha, \qquad \delta_\xi\overline{Y} \;=\; \sqrt{2}\,\bar{\xi}_{\dot\alpha}\bar{\eta}^{\dot\alpha}, \tag{2.6}$$

$$\delta_\xi\eta_\alpha = \sqrt{2}\xi_\alpha\,(H + i\partial C) + i\sqrt{2}(\bar{\xi}\bar{\sigma}^m\epsilon)_\alpha\partial_m Y, \tag{2.7}$$

$$\delta_\xi\bar{\eta}^{\dot\alpha} = \sqrt{2}\bar{\xi}^{\dot\alpha}\,(H - i\partial C) + i\sqrt{2}(\xi\sigma^m\epsilon)^{\dot\alpha}\partial_m\overline{Y}, \tag{2.8}$$

$$\delta_\xi H = \frac{i}{\sqrt{2}}(\bar{\xi}\bar{\sigma}^m\partial_m\eta) + \frac{i}{\sqrt{2}}(\xi\sigma^m\partial_m\bar{\eta}). \tag{2.9}$$

Taking care of the overall R-weights of Y and $\overline{Y}$, invariant component field Lagrangians may be obtained from ("D-term integration")

$$\mathcal{L}_{Y\overline{Y}} = \int d^2\theta d^2\bar{\theta}\,(Y\overline{Y}) = -\partial_m Y\partial^m\overline{Y} + \frac{i}{2}(\partial_m\eta\sigma^m\bar{\eta} - \eta\sigma^m\partial_m\bar{\eta}) + H^2 + \partial C^2, \tag{2.10}$$

the kinetic Lagrangian density and ("F-term integration")

$$\int d^2\theta\,Y + \int d^2\bar{\theta}\,\overline{Y} = 2H(x), \tag{2.11}$$

giving rise to the H-term referred to above.

Let us consider, as a very simple example, the coupling to a single chiral superfield[3], ϕ, of vanishing R-weight, *i.e.* adding a kinetic density

$$\int d^2\theta d^2\bar{\theta}\,\phi\bar{\phi},$$

and generalizing (2.11) to

$$\int d^2\theta\,Y\,U(\phi) + \int d^2\bar{\theta}\,\overline{Y}\,\bar{U}(\bar{\phi}),$$

with $U(\phi), \bar{U}(\bar{\phi})$ at most quadratic in the renormalizable case.

[3]Component fields for ϕ are $\phi| = A(x), D_\alpha\phi| = \sqrt{2}\chi_\alpha(x), D^2\phi| = -4F(x)$, likewise for $\bar{\phi}$.

In fact, using the explicit solutions of the constraints (2.1, 2.2)

$$Y = -4\bar{D}^2\ \Omega, \quad \overline{Y} = -4D^2\ \Omega, \tag{2.12}$$

with Ω the real unconstrained pre-potential (undetermined up to a linear superfield pre-gauge transformation) and employing integration by parts in superspace, the complete action density may be written as a pure D-term integration

$$\int d^2\theta d^2\bar{\theta}\ \left[Y\overline{Y} + \phi\bar{\phi} + 16\Omega \left(U(\phi) + \bar{U}(\bar{\phi}) \right) \right], \tag{2.13}$$

with suitable superfield equations of motion.

3 A simple model

In this section, we consider a particular combination of the 3-form and a chiral multiplet in choosing $U(\phi) = \alpha + \mu\phi$, $\mu \in \mathbb{R}$, giving rise to the superfield action density:

$$\int d^2\theta d^2\bar{\theta} \left(\phi\bar{\phi} + Y\overline{Y} \right) + \int d^2\theta \left(\alpha + \mu\phi \right) Y + \int d^2\bar{\theta} \left(\bar{\alpha} + \mu\bar{\phi} \right) \overline{Y}. \tag{3.1}$$

At the component field level, this action contains the kinetic terms for $A, \bar{A}, \chi, \bar{\chi}$ (chiral multiplet) and $Y, \overline{Y}, \eta, \bar{\eta}$ (3-form multiplet), mixing terms of these with $F, \bar{F}$ (chiral multiplet), H (3-form multiplet) and, last but not least, the terms containing ∂C, the field-strength of the 3-index antisymmetric gauge potential.

In many cases, in supersymmetric field theories, *elimination of auxiliary fields* means rather diagonalization in terms of non propagating fields (no derivative terms in the action density) with trivial algebraic equations of motion. In the case at hand, this can be done easily for the part of the action density containing $F, \bar{F}$ and H, yielding

$$\begin{aligned} \mathcal{L} \ = \ & -\partial^m A\, \partial_m \bar{A} - \frac{i}{2} \left(\chi\sigma^m \partial_m \bar{\chi} + \bar{\chi}\bar{\sigma}^m \partial_m \chi \right) - \partial_m Y \partial^m \overline{Y} - \frac{i}{2} (\eta\sigma^m \partial_m \bar{\eta} + \bar{\eta}\bar{\sigma}^m \partial_m \eta) \\ & -\mu\ (\chi\eta + \bar{\chi}\bar{\eta}) - |\,\mu\, Y|^2 - |\alpha + \mu\, A|^2 + \left[\partial C + \frac{i}{2} \left(\alpha - \bar{\alpha} + \mu\, (A - \bar{A}) \right) \right]^2 \\ & + \mathcal{F}\,\overline{\mathcal{F}} + \mathcal{H}\mathcal{H}, \end{aligned} \tag{3.2}$$

with diagonalized auxiliary fields

$$\mathcal{F} = F + \mu\overline{Y}, \quad \overline{\mathcal{F}} = \bar{F} + \mu Y, \quad \mathcal{H} = H + \frac{1}{2}(\alpha + \bar{\alpha}) + \frac{\mu}{2}(A + \bar{A}). \tag{3.3}$$

The complex scalar $Y, \overline{Y}$ satisfies a Klein-Gordon equation with mass μ, the Weyl spinors $\eta, \bar{\chi}$, combine into a Dirac spinor of the same mass. The equations of motion for the fields $A, \bar{A}, C_{klm}$ are most conveniently written using $A = A_1 + iA_2$, $\alpha = \alpha_1 + i\alpha_2$, so that

$$\begin{aligned} \Box A_1 - \mu^2 \left(A_1 + \frac{\alpha_1}{\mu} \right) &= 0, & (3.4) \\ \Box A_2 - \mu \partial C &= 0, & (3.5) \\ \partial_m \left(\partial C - \mu\, A_2 \right) &= 0. & (3.6) \end{aligned}$$

The last equation is compatible with a constant $K_2 = \partial C - \mu\, A_2$, giving rise to a shifted Klein-Gordon equation for A_2

$$(\Box - \mu^2) \left(A_2 + \frac{\alpha_2 + K_2}{\mu} \right) = 0. \tag{3.7}$$

We would like to stress that these features arise necessarily in the context of models dealing with gaugino condensation.

4 The $X - Y$ Correspondence

Independently of supersymmetry, the 3-index antisymmetric gauge potential C_{klm} has been employed in the context of the cosmological constant problem [8], [9]. The derivative quadratic action density is proportional to $(\partial C)^2$. This density can be related to a constant considering the density

$$X^2 + X\partial C$$

with $X(x)$ a real field. Varying with respect to X and substituting back reproduces $(\partial C)^2$. On the other hand, varying with respect to C_{klm} implies $\partial_m X = 0$, *i.e.* X a constant.

This mechanism can be extended to the supersymmetric case, *e.g.* the 3-form multiplet. Here we consider the combination

$$\int d^2\theta d^2\bar{\theta}\Big[-X\overline{X} - XY - \overline{Y}\overline{X} + 16\Omega\left(U(\phi) + \bar{U}(\bar{\phi})\right) + \phi\bar{\phi}\Big]. \tag{4.1}$$

$X, \overline{X}$ is a complex unconstrained superfield, $Y, \overline{Y}$, the 3-form superfield introduced above and Ω its unconstrained real pre-potential. $\phi, \bar{\phi}$ are considered as spectator superfields. Varying with respect to $X, \overline{X}$ just implies $X = \overline{Y}$, $\overline{X} = Y$ and one recovers (2.13) upon substitution. As to variation with respect to the 3-form multiplet we shall use the solution (2.12) of the constraints and integration by parts in superspace to arrive at

$$\int d^2\theta d^2\bar{\theta}\Big[-X\overline{X} + 4\Omega\left(\bar{D}^2X + D^2\overline{X} + 4U(\phi) + 4\bar{U}(\bar{\phi})\right) + \phi\bar{\phi}\Big], \tag{4.2}$$

where Ω may be considered as a Lagrange multiplier superfield giving rise to a constraint

$$\bar{D}^2X + D^2\overline{X} + 4U(\phi) + 4\bar{U}(\bar{\phi}) = 0, \tag{4.3}$$

that can be separated into two constraints

$$\bar{D}^2X = -4U(\phi) - 4K, \qquad D^2\overline{X} = -4\bar{U}(\bar{\phi}) - 4\bar{K}, \tag{4.4}$$

related by a constant $K = -\bar{K} = iK_2$, which might be absorbed in a redefinition of $U(\phi), \bar{U}(\bar{\phi})$. In other words, in supersymmetry, the analogue of the constant mentioned above (in the non supersymmetric case) is given by a complex superfield, $X, \overline{X}$.

The component field action is then obtained from

$$\int d^2\theta d^2\bar{\theta}\Big[-X\overline{X} + \phi\bar{\phi}\Big]. \tag{4.5}$$

In the case $U = 0$, this multiplet has been presented in [5]. We shall call it *non-minimal* in what follows and use the term *coupled non-minimal* in the case of non vanishing U, to be discussed in the next section.

5 The coupled non-minimal multiplet

The superfield constraints (4.4) determine a multiplet of 12 bosonic and 12 fermionic component field degrees of freedom, identified as usual by successive applications of covariant spinor derivatives. We define the component fields contained in X as

$$\begin{aligned} X| &= X, \qquad D_\alpha X| = \sqrt{2}\,\psi_\alpha, \qquad \bar{D}^{\dot\alpha} X| = -\sqrt{2}\,\bar\omega^{\dot\alpha}, \\ \bar{D}^{\dot\alpha} D_\alpha X| &= V_\alpha{}^{\dot\alpha}, \quad \bar{D}^{\dot\alpha} D^2 X| = -4\bar\rho^{\dot\alpha}, \quad D^2 X| = -4E. \end{aligned} \tag{5.6}$$

Observe that the $\bar\theta^2$ component is given in terms of A, Cf.(4.4). For $\overline{X}$ we define similarly

$$\begin{aligned} \overline{X}| &= \overline{X}, \qquad D_\alpha \overline{X}| = -\sqrt{2}\,\omega_\alpha, \qquad \bar{D}^{\dot\alpha} \overline{X}| = \sqrt{2}\,\bar\psi^{\dot\alpha}, \\ D_\alpha \bar{D}^{\dot\alpha} \overline{X}| &= \bar{V}_\alpha{}^{\dot\alpha}, \quad D_\alpha \bar{D}^2 \overline{X}| = -4\rho_\alpha, \quad \bar{D}^2 \overline{X}| = -4\bar{E}. \end{aligned} \tag{5.7}$$

Projecting $\frac{1}{16} D\bar{D}^2 D(-X\overline{X})$ to lowest components gives the canonical component field action density[4]

$$\begin{aligned} \mathcal{L}_X = &-\partial_m X \partial^m \overline{X} - \frac{i}{2}\left(\omega\sigma^m\partial_m\,\bar\omega + \bar\omega\bar\sigma^m\partial_m\,\omega\right) \\ &-|\,U|^2 - U'\left(\overline{X}F + \omega\chi\right) - \bar{U}'\left(X\bar{F} + \bar\omega\bar\chi\right) + \frac{1}{2}\,\overline{X}U''(\chi\chi) + \frac{1}{2}\,X\bar{U}''(\bar\chi\bar\chi) \\ &-\frac{1}{2}V_m\bar{V}^m - E\bar{E} - \frac{i}{2}\psi\left(\sigma^m\partial_m\,\bar\psi + i\sqrt{2}\rho\right) - \frac{i}{2}\bar\psi\left(\bar\sigma^m\partial_m\,\psi + i\sqrt{2}\bar\rho\,\right), \end{aligned} \tag{5.8}$$

describing a complex scalar X and a Majorana spinor ω as physical fields. Auxiliary fields consist of a complex scalar E, a complex vector V_m, and 2 Majorana spinors ψ, ρ. This action density is invariant under supersymmetry transformations:

$$\begin{aligned} &\delta_\xi X = \sqrt{2}\,(\xi\psi - \bar\xi\bar\omega), \qquad \delta_\xi \overline{X} = \sqrt{2}\,(\bar\xi\bar\psi - \xi\omega), \\ &\delta_\xi\psi_\alpha = \sqrt{2}E\,\xi_\alpha - \frac{1}{\sqrt{2}}V_m\,(\bar\xi\bar\sigma^m\epsilon)_\alpha, \qquad \delta_\xi\bar\psi^{\dot\alpha} = \sqrt{2}\bar{E}\,\bar\xi^{\dot\alpha} - \frac{1}{\sqrt{2}}\,\bar{V}_m(\xi\sigma^m\epsilon)^{\dot\alpha}, \\ &\delta_\xi\bar\omega^{\dot\alpha} = -\sqrt{2}U\bar\xi^{\dot\alpha} - \frac{1}{\sqrt{2}}(V_m + 2i\partial_m X)(\xi\sigma^m\epsilon)^{\dot\alpha}, \\ &\delta_\xi\omega_\alpha = -\sqrt{2}\bar{U}\xi_\alpha - \frac{1}{\sqrt{2}}\,(\bar{V}_m + 2i\partial_m\overline{X})(\bar\xi\bar\sigma^m\epsilon)_\alpha, \end{aligned}$$

[4]Primes indicate derivatives with respect to A or $\bar{A}$, as the case may be.

$$
\begin{aligned}
&\delta_\xi\ V_m = (\xi\sigma_m\bar\rho) + \sqrt{2}i\ (\xi\sigma_n\bar\sigma_m\partial^n\psi - \bar\xi\bar\sigma_m\sigma_n\partial^n\bar\omega) - (\bar\xi\bar\sigma_m\chi)U',\\
&\delta_\xi\ \bar V_m = (\bar\xi\bar\sigma_m\rho) + \sqrt{2}i\ (\bar\xi\bar\sigma_n\sigma_m\partial^n\bar\psi - \xi\sigma_m\bar\sigma_n\partial^n\omega) - (\xi\sigma_m\bar\chi)\bar U',\\
&\delta_\xi\bar\rho^{\dot\alpha} = 2i\ \ \partial_m E\ (\xi\sigma^m\epsilon)^{\dot\alpha} + (2i\partial_m V^m - 2\Box X - U''\chi\chi + 2U'F)\ \bar\xi^{\dot\alpha},\\
&\delta_\xi\rho_\alpha = 2i\ \partial_m\bar E\ (\bar\xi\bar\sigma^m\epsilon)_\alpha\ + (2i\partial_m\bar V^m - 2\Box\overline{X} - \bar U''\bar\chi\bar\chi + 2\bar U'\bar F)\ \xi_\alpha\ ,\\
&\delta_\xi E = \bar\xi\,\bar\rho, \qquad \delta_\xi\bar E = \xi\,\rho,\\
&\delta_\xi\bar D^2 X = -4\sqrt{2}U'(\xi\chi), \qquad \delta_\xi D^2\overline{X} = -4\sqrt{2}\bar U'(\bar\xi\bar\chi).
\end{aligned}
\tag{5.9}
$$

Adding the kinetic Lagrangian for ϕ

$$
\mathcal{L}_{\mathcal{S}} \ = \ -\partial^m A\,\partial_m\bar A - \frac{i}{2}\,(\chi\sigma^m\partial_m\ \bar\chi + \bar\chi\bar\sigma^m\partial_m\ \chi) + F\bar F, \tag{5.10}
$$

the complete Lagrangian is

$$
\begin{aligned}
\mathcal{L} \ = \ & -\partial^m A\,\partial_m\bar A - \frac{i}{2}\,(\,\chi\sigma^m\partial_m\ \bar\chi + \bar\chi\bar\sigma^m\partial_m\ \chi) - \partial_m X\partial^m\overline{X} - \frac{i}{2}\,(\,\omega\sigma^m\partial_m\ \bar\omega + \bar\omega\bar\sigma^m\partial_m\ \omega)\\
& -U'(A)\,\omega\chi - \bar U'(\bar A)\,\bar\omega\bar\chi - |U'(A)|^2 X\overline{X} - |U(A)|^2\\
& +\frac{1}{2}\,\overline{X}U''(A)(\chi\chi) + \frac{1}{2}\,X\bar U''(\bar A)(\bar\chi\bar\chi) + \mathcal{F}\,\overline{\mathcal{F}} - \frac{1}{2}V_m\bar V^m - E\bar E\\
& -\frac{i}{2}\psi\left(\sigma^m\partial_m\ \bar\psi + i\sqrt{2}\rho\right) - \frac{i}{2}\bar\psi\left(\bar\sigma^m\partial_m\ \psi + i\sqrt{2}\bar\rho\ \right),
\end{aligned}
\tag{5.11}
$$

with

$$
\mathcal{F} = F - X\bar U', \quad \overline{\mathcal{F}} = \bar F - \overline{X}U', \tag{5.12}
$$

and exhibiting the general scalar potential

$$
\mathcal{V} = |U'(A)|^2 X\overline{X} + |U(A)|^2. \tag{5.13}
$$

In order to make contact with the simple model of section 3, we set $U(A) = \alpha + \mu A$, $\bar U(\bar A) = \bar\alpha + \mu\bar A$. Then (5.11) describes two complex scalar fields and a Dirac field with common mass μ, just like the Lagrangian (3.2). The difference between the two Lagrangians appears in the auxiliary field sector and, correspondingly, in the component field supersymmetry transformations. Moreover, it should be stressed that $Y, \overline{Y}$ represents a gauge multiplet, whereas $X, \overline{X}$ does not; this is also the case for the linear-chiral multiplet correspondence.

6 Conclusions

The main purpose of this short communication was to establish a correspondence between the 3-form multiplet and a non-minimal multiplet, in analogy to the well-known relation between the 2-form (*i.e.* linear) multiplet and the chiral multiplet. Observe that in both cases the correspondence can only be established under certain restrictive assumptions.

Although the 3-form multiplet and the non-minimal multiplet might be considered as *exotic multiplets*, they are not. As indicated in the introduction, the 3-form multiplet describes naturally the gaugino squared chiral superfield $tr\,(W^\alpha W_\alpha)$ and its complex conjugate. On the other hand, the non-minimal multiplet appears naturally in the context of the solution of the chiral superfield constraints, *i.e.* $\phi = \bar{D}^2\varphi$, $\bar{\phi} = D^2\bar{\varphi}$, in terms of unconstrained potentials $\varphi, \bar{\varphi}$, defined up to pre-gauge transformations $\varphi \rightarrow \varphi + \xi$, $\bar{\varphi} \rightarrow \bar{\varphi} + \bar{\xi}$. These superfields are themselves subject to the pre-constraints $\bar{D}^2\xi = 0$, $D^2\bar{\xi} = 0$, leaving $\phi, \bar{\phi}$ invariant.

Let us mention as well that the above-mentioned 3-form constraints appear in an intriguing way in supergravity, in the framework of $U(1)$ superspace. The chiral supergravity superfields $R, R^\dagger$ are intertwined with the vector superfield G_a through the relation $\mathcal{D}^2 R - \bar{\mathcal{D}}^2 R^\dagger = 4i\mathcal{D}^a G_a$. Remarkably enough, here, the *H-term* of $R, R^\dagger$ corresponds to a *D-term* of the $U(1)$ supergravity sector.

The emphasis of the present note was to draw attention to the basic features of the correspondence between the 3-form multiplet and the non-minimal multiplet restricting ourselves to quite elementary considerations. More involved structures as well as the corresponding supergravity couplings will be the subject of forthcoming publications.

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